Review of Chapter 1 and 2

Name: Key

Date: ____________________ Pd: _____

**What are the parts of a proof?**

1. Statement of the ***theorem*** and/or a ***diagram*** that illustrates the given information
2. A list of the ***given*** information
3. A list of what you are to ***prove***
4. A series of numbered ***statements*** that lead to what you are trying to prove
5. A series of ***reasons*** that justify each statement

**What can be used as reasons in a proof?**

1. Given information
2. Definitions
3. Properties
4. Postulates
5. Theorems that have already been proved

Determine whether the following conditional statement is true or false.
Then write the converse and determine whether the converse statement is true or false.
If either statement is false, provide a counterexample.

If $\sqrt{x}=2$, then $x^2=16$. $\boxed{\text{T}}$ F

$\sqrt{x}=2$

$x=4$ $\rightarrow$ $4^2=16$ ✓

Converse: If $x^2=16$, then $\sqrt{x}=2$. T F

$x^2=16$ CE: Let $x=-4$.

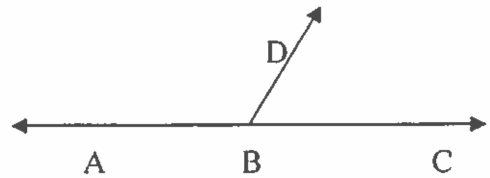
$x=\pm\sqrt{16}$ $(-4)^2=16$, but $\sqrt{-4}\neq 2$.

$x=\pm 4$ Hyp True Concl. False

Can you write a true biconditional statement from the information above?
Explain why you can or cannot. If you can, write it below.

You cannot write a biconditional statement because that requires both the original statement and the converse to be true.

Look at the diagram below.
Write the three statements that can be justified by the ***Angle Addition Postulate.***




1. $m\angle ABD + m\angle DBC = m\angle ABC$
2. $m\angle ABD + m\angle DBC = 180°$
3. $\angle ABD$ and $\angle DBC$ are supplements

**The Midpoint Theorem**

If M is the midpoint of $\overline{AB}$, then $AM = MB$, $AM = \frac{1}{2}AB$, and $MB = \frac{1}{2}AB$.

**Diagram**


Given: M is the midpoint of $\overline{AB}$.

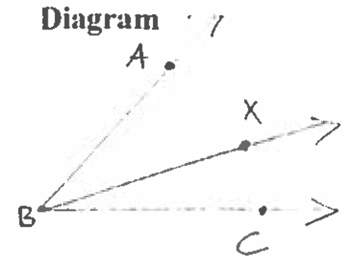
Prove: $Am = mB$, $Am = \frac{1}{2}AB$, and $mB = \frac{1}{2}AB$

| Statements | Reasons |
|---|---|
| 1. M is the midpoint of $\overline{AB}$ | Given |
| 2. $\overline{Am} \cong \overline{mB}$ | Def of midpt |
| 3. $Am = mB$ | Def. of $\cong$ segments |
| 4. $Am + mB = AB$ | Seg. Add. Post. |
| 5. $Am + Am = AB$ | Subst. Prop. of = (3→4) |
| 6. $2Am = AB$ | Dist. prop. |
| 7. $Am = \frac{1}{2}AB$ | Div. Prop. of = |
| 8. $mB = \frac{1}{2}AB$ | Subst. Prop. of = (3→7) |

**The Angle Bisector Theorem**

If $\overrightarrow{BX}$ is the bisector of $\angle ABC$, then $m\angle ABX = m\angle XBC$, $m\angle ABX = \frac{1}{2}m\angle ABC$, and $m\angle XBC = \frac{1}{2}m\angle ABC$.

**Diagram**



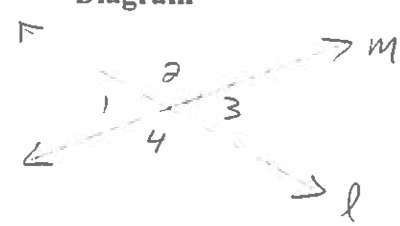

Given: $\overrightarrow{BX}$ is the bisector of $\angle ABC$

Prove: $m\angle ABX = m\angle XBC$, $m\angle ABX = \frac{1}{2}m\angle ABC$, and $m\angle XBC = \frac{1}{2}m\angle ABC$

| Statements | Reasons |
|---|---|
| 1. $\overrightarrow{BX}$ is the bisector of $\angle ABC$ | Given |
| 2. $\angle ABX \cong \angle XBC$ | Def. of $\angle$ bisector |
| 3. $m\angle ABX = m\angle XBC$ | Def. of $\cong$ $\angle$s |
| 4. $m\angle ABX + m\angle XBC = m\angle ABC$ | $\angle$ Add. Post. |
| 5. $m\angle ABX + m\angle ABX = m\angle ABC$ | Subst. Prop. of = $(3 \to 4)$ |
| 6. $2m\angle ABX = m\angle ABC$ | Dist. Prop. |
| 7. $m\angle ABX = \frac{1}{2}m\angle ABC$ | Div. Prop. of = |
| 8. $m\angle XBC = \frac{1}{2}m\angle ABC$ | Subst. Prop. of = $(3 \to 7)$ |

**Vertical Angle Theorem** – Vertical angles are congruent.

**Diagram**




Given: $l$ and $m$ intersect

Prove: $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

| Statements | Reasons |
|---|---|
| 1. $m\angle 1 + m\angle 2 = 180°$, $m\angle 2 + m\angle 3 = 180°$ | $\angle$ Add Post |
| 2. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ | Trans. Prop. of = |
| 3. $m\angle 2 = m\angle 2$ | Refl. Prop. of = |
| 4. $m\angle 1 = m\angle 3$ | Subtr. Prop. of = $(2-3)$ |
| 5. $\angle 1 \cong \angle 3$ | Def. of $\cong$ $\angle$s |

Likewise, $\angle 2 \cong \angle 4$.

**Right Angles Theorem** – All right angles are congruent.

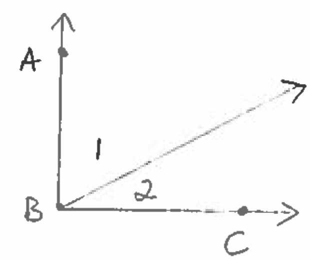
Given: $\angle A$ and $\angle B$ are right $\angle s$

Prove: $\angle A \cong \angle B$

| Statements | Reasons |
|---|---|
| 1. $\angle A$ and $\angle B$ are right $\angle s$ | Given |
| 2. $m\angle A = 90°$, $m\angle B = 90°$ | Def. of right $\angle s$ |
| 3. $m\angle A = m\angle B$ | Trans. Prop. of = |
| 4. $\angle A \cong \angle B$ | Def. of $\cong \angle s$ |

**Theorem:** If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.

**Diagram**



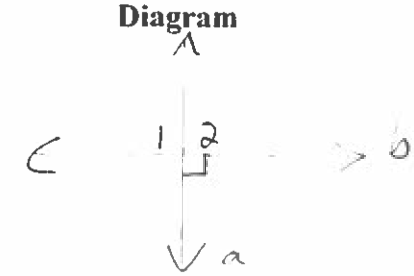

Given: $\overline{AB} \perp \overline{BC}$

Prove: $\angle 1$ and $\angle 2$ are complementary

| Statements | Reasons |
|---|---|
| 1. $\overline{AB} \perp \overline{BC}$ | Given |
| 2. $m\angle ABC = 90°$ | Def. of $\perp$ |
| 3. $m\angle 1 + m\angle 2 = m\angle ABC$ | $\angle$ Add. Post. |
| 4. $m\angle 1 + m\angle 2 = 90°$ | Trans. Prop. of = |
| 5. $\angle 1$ and $\angle 2$ are complementary | Def. of complementary $\angle s$ |

**Theorem:** If two lines are perpendicular, then they form congruent adjacent angles.

**Diagram**



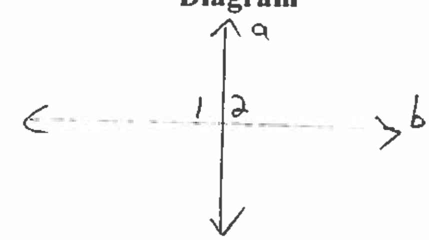

Given: $a \perp b$

Prove: $\angle 1 \cong \angle 2$

| Statements | Reasons |
|---|---|
| 1. $a \perp b$ | Given |
| 2. $\angle 1$ and $\angle 2$ are right $\angle s$ | Def. of $\perp$ |
| 3. $\angle 1 \cong \angle 2$ | Right $\angle$ thrm |

**Theorem:** If two lines form congruent adjacent angles, then the lines are perpendicular.

**Diagram**




Given: $\angle 1 \cong \angle 2$

Prove: $a \perp b$

| Statements | Reasons |
|---|---|
| 1. $\angle 1 \cong \angle 2$ | Given |
| 2. $m\angle 1 = m\angle 2$ | Def. of $\cong$ $\angle s$ |
| 3. $m\angle 1 + m\angle 2 = 180°$ | $\angle$ Add. Post. |
| 4. $m\angle 1 + m\angle 1 = 180°$ | Subst. Prop. of = (2→3) |
| 5. $2m\angle 1 = 180°$ | Dist. Prop. |
| 6. $m\angle 1 = 90°$ | Div. Prop. of = |
| 7. $a \perp b$ | Def. of $\perp$ |

Since these two theorems are converses of each other, write a biconditional statement.

Two lines are $\perp$ iff they form $\cong$ adj. $\angle s$.

The proofs for the Congruent Complements and the Congruent Supplements theorems follow very similar reasoning. Make sure you can prove both.

**Congruent Supplements Thrm** -- If two angles are supplements of the same or congruent angles, then the two angles are congruent.

Given: $\angle 1$ and $\angle 2$ are supplements, $\angle 2$ and $\angle 3$ are supplements

Prove: $\angle 1 \cong \angle 3$

| Statements | Reasons |
|---|---|
| 1. $\angle 1$ is supp. to $\angle 2$, $\angle 2$ is supp. to $\angle 3$ | Given |
| 2. $m\angle 1 + m\angle 2 = 180^\circ$, $m\angle 2 + m\angle 3 = 180^\circ$ | Def. of supp. $\angle$s |
| 3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ | Trans. Prop. of = |
| 4. $m\angle 2 = m\angle 2$ | Refl. prop. of = |
| 5. $m\angle 1 = m\angle 3$ | Subtr. Prop. of = (3-4) |
| 6. $\angle 1 \cong \angle 3$ | Def. of $\cong \angle$s |

The proofs for the Congruent Complements Converse and the Congruent Supplements Converse follow very similar reasoning. Make sure you can prove both.

**Congruent Complements Converse** – If two angles are congruent, then they are complementary to the same or congruent angles.

Given: $\angle 1 \cong \angle 2$, $\angle 1$ is comp. to $\angle 3$

Prove: $\angle 2$ is comp. to $\angle 3$

| Statements | Reasons |
|---|---|
| 1. $\angle 1 \cong \angle 2$, $\angle 1$ is comp. to $\angle 3$ | Given |
| 2. $m\angle 1 = m\angle 2$ | Def. of $\cong \angle$s |
| 3. $m\angle 1 + m\angle 3 = 90^\circ$ | Def. of comp. $\angle$s |
| 4. $m\angle 2 + m\angle 3 = 90^\circ$ | Subst. Prop. of = $(2 \rightarrow 3)$ |
| 5. $\angle 2$ is comp. to $\angle 3$ | Def. of comp. $\angle$s |